Statistical Engine Misfire Detection

Alexander Stotsky

Abstract— **New recursive filtering algorithms for misfire detection based on the trigonometric interpolation method are proposed for spark ignition automotive engines. The technique improves the performance of the filtering algorithms allowing a flexible choice of the size of the moving window. Correction algorithms are introduced for the recursive trigonometric interpolation method that ensure the robustness with respect to round-off errors which are present in the finite precision implementation environment. New real-time statistical algorithms based on a hypothesis testing for a misfire detection are proposed. Statistical decision making mechanism allows to make a misfire detection with a certain significance level with automatically selected sample size depending on the signal quality that in turn improves the robustness of the misfire detection algorithm.** [1]

Keywords: Fourier analysis , Filtering techniques , Misfire, Hypotheses, Six Sigma, Spark Ignition Engines

I. INTRODUCTION

Misfire is the state of an engine where the combustion does not occur due to the errors in fueling or ignition. As a consequence, such misfires affect long term performance of the exhaust emission control system. The misfiries cause changes in the crankshaft rate of rotation, because the misfired cylinder is not able to provide the torque. Engine misfire diagnostic functions are based on monitoring of the cylinder individual fluctuations of the high resolution engine speed signal or a passage time between subsequent teeth on a crankwheel. The high resolution engine speed signal is calculated as a ratio of the length of the angular segment on the crankwheel and the passage time for this segment. The passage time becomes less as the rotational speed rises, thereby time interval errors rise. Moreover, low frequency oscillations from the powertrain and high frequency oscillations due to the crankshaft torsion, together with vibrations induced by the road, act as disturbances on the crankshaft. These disturbances influence directly the performance of the engine speed signal and consequently the torque monitoring and misfire diagnostic functions. Recursive DFT (Discrete Fourier Transformation) method in the window of a certain size w moving in time can be used for filtering at the engine firing frequency [2],[3]. However, the orthogonality condition for the trigonometric polynomials in certain interval is the main restriction of the application of the DFT method. This is turn imposes restrictions on the window size w. If the orthogonality condition is violated, then the implementation of the DFT method (in this case a better name is a trigonometric interpolation method) requires a matrix inversion, as it is usual for least-squares fitting, and this in turn, makes the method computationally expensive. Algorithms proposed in the present paper allow to make a trigonometric interpolation of the engine speed data for any window size. This is turn allows to improve the performance of the misfire diagnostic function. Recursive and computationally efficient version of the trigonometric interpolation method is developed.

Limited precision effects might severely deteriorate the performance of the recursive trigonometric interpolation method. The accumulation of round-off errors in a finite precision engine control implementation environment limits the trigonometric interpolation and hence the misfire detection performance. In the present paper the correction algorithms are proposed for correction of the estimates obtained by the recursive trigonometric interpolation method.

The misfire detection approach proposed in the present paper is based on a monitoring of the amplitudes at two frequencies. The first one is the engine firing frequency. The second amplitude is the amplitude of the component of the engine speed with a period of 720^o. Since the misfiring cylinder is active every 720^o of the crankshaft rotation a torque drop associated with the misfire occurs every 720^o generating a component of the engine speed with a period of 720^o. A torque behaviour associated with the misfire occurs once every two cycles of crankshaft rotation or one-half cycles per crankshaft revolution. This is typically referred as a half-order behaviour and the component of the engine speed with the period of 720^o is referred as a half engine order component. The location on the engine cycle of the minimum of this component of the engine speed shows which cylinder is misfiring. For example, the firing sequence of the six cylinder prototype engine is the following 1-5-3-6-2-4. Figure 1 shows two misfires instead of two neighboring combustions, i.e., in the first and fifth cylinders. Figure 1 shows that the misfires can be detected by monitoring the amplitude not only at the engine combustion frequency and also the amplitude at the half-order frequency whereby the phase of the half-order frequency component shows which cylinder is misfiring. Here and below, an unusual definition is used, and under the term 'amplitude' the difference between maximal and minimal values is understood.

The amplitudes are hypothesis tested for a misfire detection. *One Sample T-test* (the name is carried over from [1]) which compares the average value of the amplitude with the target value is used for a misfire detection at the combustion frequency. *Two Sample T-test* which compares the average values of the largest and the next largest amplitudes at the half-order frequency is used for a misfire detection at the half-order frequency. The algorithm is divided into two

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[1]This work was done within the Volvo Six Sigma programme

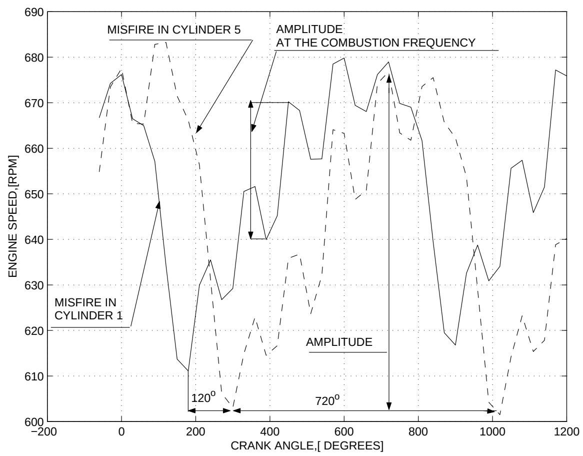

Fig. 1. Two engine cycles are plotted in the events of a misfire in the fisrt and fifth cylinders. Engine is operating at idle. Engine speed signal in the event of a misfire in the first cylinder is plotted with a solid line. Engine speed signal in the event of a misfire in the fifth cylinder is plotted with a dashed line.

steps in both cases. In the first step the hypotheses are tested at each step of the moving window of a minimal size, where the value of $t - statistic$ is compared with the value in the Student distribution look-up table for the degrees of freedom corresponding to the minimal size of the window. The window is defined here in terms of a number of engine cycles. If the value of $t - statistic$ is greater than the value in the Student distribution look-up table the null hypotheses are rejected detecting a misfire. If the value of $t - statistic$ is less than the value in the Student distribution look-up table then window size is increased until the value of $t - statistic$ is greater than the value in the Student distribution look-up table or the window size reaches its maximal value. To this end the next step is taken. Since both $t - statistic$ and the values in the Student distribution look-up table depend on the size of the moving window the values in the Student distribution look-up table for a certain significance level are approximated by a polynomial of a certain order. The equation for $t - statistic$ which is equal to the polynomial approximation of the Student distribution look-up table is solved with respect to the window size, indicating the minimal window size for which the hypotheses can be tested and a misfire can be detected. This approach allows a reliable misfire detection for amplitude signals of different quality (for example for new and aged engine) via a proper selection of the window size (degrees of freedom).

A Volvo passenger car equipped with a six cylinder prototype engine was used in the experiments. Algorithms are implemented in MATLAB[2] and applied to the measured data collected from the experimental vehicle.

The contributions of the present paper can be summarized as follows: a) new recursive filtering algorithms for misfire detection based on the trigonometric interpolation method, b) new statistical algorithms based on a hypothesis testing for a misfire detection.

[2]MATLAB is a registered mark of the Mathworks, Inc of Natick, MA

II. Recursive Trigonometric Interpolation Algorithms

A. Problem Statement

Suppose that there is a set of the crank angle synchronized measurements of the engine speed ω_k, $k = 1, 2...$, measured at the following points $x_k = k\Delta$, where Δ is a step size. Suppose that the engine speed signal can exactly be approximated by the trigonometric polynomial as follows:

$$\hat{\omega}_k = \varphi_k^T \theta_k, \tag{1}$$

$$\theta_k^T = [a_{0k} \quad a_{q1k} \quad b_{q1k} \quad a_{q2k} \quad b_{q2k}, \\ ..., a_{qnk} \quad b_{qnk}], \tag{2}$$

$$\varphi_k^T = [1 \quad cos(q_1 x_k) \quad sin(q_1 x_k) \quad cos(q_2 x_k) \\ sin(q_2 x_k), ..., cos(q_n x_k) \quad sin(q_n x_k)] \tag{3}$$

where θ_k is the vector of the adjustable parameters and φ_k is the regressor, $q = q_1, ..., q_n$ are the frequencies, a_{0k}, a_{qk} and b_{qk} are the coefficients which should be found, $\hat{\omega}_k$ is the estimate of the engine speed ω_k. Assume that measured engine speed signal can be presented as follows:

$$\omega_k = \varphi_k^T \theta_*, \tag{4}$$

where θ_* is the vector of true parameters,

$$\theta_*^T = [a_{0*} \quad a_{q1*} \quad b_{q1*} \quad a_{q2*} \quad b_{q2*}, ..., a_{qn*} \quad b_{qn*}], \tag{5}$$

and a_{0*}, a_{q*} and b_{q*} are constant unknown coefficients.

Introducing a moving window of a size w the measured engine speed signal ω_k is approximated by (1) in the least squares sense. The error to be minimized at every step is as follows:

$$E_k = \sum_{i=k-(w-1)}^{i=k} (\omega_i - \hat{\omega}_i)^2, \qquad k \geq w \tag{6}$$

B. Recursive Algorithms for Trigonometric Interpolation

The vector of adjustable parameters θ_k at step k which minimizes the performance index (6) can be computed as follows:

$$\theta_k = [\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T]^{-1} \sum_{i=k-(w-1)}^{i=k} \varphi_i \omega_i \tag{7}$$

Notice that, if the orthogonality condition for trigonometric polynomials is satisfied for a certain window size the matrix $\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T$ becomes a diagonal matrix. This matrix is easily invertable and the vector of adjustable parameters θ_k is the vector of the Fourier coefficients. Therefore, the DFT method can be seen as a special case of the trigonometric interpolation method described below.

The vector of adjustable parameters at step $k - 1$ of the moving window can be computed as follows:

$$\theta_{k-1} = [\sum_{i=k-w}^{i=k-1} \varphi_i \varphi_i^T]^{-1} \sum_{i=k-w}^{i=k-1} \varphi_i \omega_i \quad (8)$$

In step k of the moving window new data ω_k, φ_k enters the window and ω_{k-w}, φ_{k-w} leaves the window.

The vector of adjustable parameters θ_k can be presented as follows:

$$\begin{aligned}\theta_k &= [(\sum_{i=k-w}^{i=k-1} \varphi_i \varphi_i^T) - \varphi_{k-w}\varphi_{k-w}^T + \varphi_k \varphi_k^T]^{-1} \\ & [(\sum_{i=k-w}^{i=k-1} \varphi_i \omega_i) + \varphi_k \omega_k - \varphi_{k-w}\omega_{k-w}] \quad (9)\end{aligned}$$

Applying matrix inversion relation to (9) straightforward calculations show that the vector of the adjustable parameters at step k can be computed via the vector of adjustable parameters at step $k-1$ as follows:

$$\begin{aligned}&\theta_{rk} = (I - \frac{A_{k-1}\varphi_k \varphi_k^T}{1+\varphi_k A_{k-1}\varphi_k^T} \\ &(\theta_{r(k-1)} + \frac{\Gamma_{k-1}\varphi_{k-w}\varphi_{k-w}^T \theta_{r(k-1)}}{1-\varphi_{k-w}^T \Gamma_{k-1}\varphi_{k-w}})) \\ &+\Gamma_k(\varphi_k \omega_k - \varphi_{k-w}\omega_{k-w}) \quad (10)\end{aligned}$$

where

$$\Gamma_k = A_{k-1}(I - \frac{\varphi_k \varphi_k^T A_{k-1}}{1+\varphi_k A_{k-1}\varphi_k^T}) \quad (11)$$

$$A_{k-1} = \Gamma_{k-1}(I + \frac{\varphi_{k-w}\varphi_{k-w}^T \Gamma_{k-1}}{1-\varphi_{k-w}^T \Gamma_{k-1}\varphi_{k-w}}) \quad (12)$$

where θ_{rk} is a recursive estimate of the parameter vector θ_k, I is the identity matrix and Γ_{k-1} is a recursive estimate of $[\sum_{i=k-w}^{i=k-1} \varphi_i \varphi_i^T]^{-1}$ and Γ_k is a recursive estimate of $[\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T]^{-1}$.
The elements of the regressor φ_i, $i \geq 3$ are recursively calculated via the following Chebyshev's three term recurrence relations

$$\begin{aligned}&\varphi_i = 2d_q \quad * \quad \varphi_{i-1} - \varphi_{i-2} \quad (13)\\ &\varphi_i^T = [1 \quad cos(q_1 i\Delta) \quad sin(q_1 i\Delta) \quad cos(q_2 i\Delta) \\ &\quad sin(q_2 i\Delta), ..., cos(q_n i\Delta) \quad sin(q_n i\Delta)] \quad (14)\\ &d_q^T = [1 \quad cos(q_1\Delta) \quad cos(q_1\Delta) \quad cos(q_2\Delta) \\ &\quad cos(q_2\Delta), ..., cos(q_n\Delta) \quad cos(q_n\Delta)] \quad (15)\end{aligned}$$

where Δ is the sampling step, and '$*$' denotes element-wise vector multiplication, and index i is equal to k, $(i = k)$ and index i is equal to $k-w$, $(i = k-w)$, $\quad k \geq (w+3)$ for the recursive computations of φ_k and φ_{k-w} respectively in (10)-(12). For the recursive computations of the regressor over the whole window which are required in the first step of the moving window $k = w$ and for calculations of the approximation error (6) the index i is defined as follows $i = k-w+p$, where $p = 3, ..., w$.
Notice that in the absence of round-off errors $\theta_{rk} \equiv \theta_k$. Robustness and correction of the recursive algorithms (10), (11) with respect to the round-off error accumulation is discussed in the next subsection.

C. *Correction of the Recursive Algorithms for Round-Off Errors*

On-board implementation of the recursive trigonometric interpolation algorithms is done in ECU (Engine Control Unit) in the finite precision environment, where round-off errors are recursively accumulated. This makes the recursive trigonometric interpolation algorithm unsuitable for continuous use without correction. Correction algorithms proposed in this subsection use Newton's algorithms for correction of the estimates obtained by recursive trigonometric interpolation method. The parameter vector θ_{rk} and the inverse of the 'information matrix' Γ_k obtained by using the recursive least-squares algorithm (10), (11) are used as the initial states for the correction algorithms.

1) Correction of θ_{rk} : Suppose that the recursive algorithm for calculation of θ_{rk} (10) accumulated rounding errors so that (7) (with θ_{rk} substituted instead of θ_k) is not valid. Consider the following algorithm for correction of θ_{rk}.

$$\theta_{cj} = \theta_{c(j-1)} - \Gamma_k e_{j-1} \quad (16)$$

where θ_{cj} is the correction of the parameter vector θ_{rk}, $e_{j-1} = (\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T)\theta_{c(j-1)} - \sum_{i=k-(w-1)}^{i=k} \varphi_i \omega_i$, $j = 1, 2, 3...$ with the initial value of $\theta_{c0} = \theta_{rk}$ and Γ_k calculated with (11). The estimation error $\tilde{\theta}_{cj} = \theta_{cj} - \theta_k$ where θ_k is defined in (7) satisfies the following equation:

$$\tilde{\theta}_{cj} = (I - \Gamma_k[\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T])\tilde{\theta}_{cj-1} \quad (17)$$

Notice that Γ_k represents a recursive estimate of $[\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T]^{-1}$. In the absence of the rounding errors $\Gamma_k \equiv [\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T]^{-1}$, $\tilde{\theta}_{cj} \equiv 0$ and $\theta_{cj} \equiv \theta_k$. Since rounding errors also have an impact on the recursive estimate of Γ_k calculated with (11), $\Gamma_k \neq [\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T]^{-1}$ and $\tilde{\theta}_{cj} \to 0$ as $j \to \infty$ if the eigenvalues of the following matrix $(I - \Gamma_k[\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T])$ are located inside of the unit circle.

2) Correction of Γ_k : As it was mentioned above rounding errors have also impact on the matrix Γ_k calculated with (11). The following iterative method similar to the method described above can also be applied for the correction of the elements of the matrix Γ_k.

$$\Gamma_{cj} = \Gamma_{c(j-1)} + \Gamma_{c(j-1)} F_{j-1} \quad (18)$$

where Γ_{cj} is the correction of the matrix Γ_k, $F_{j-1} = I - [\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T]\Gamma_{c(j-1)}$ with the initial condition $\Gamma_{c0} = \Gamma_k$, $j = 1, 2, ...$ Straightforward calculations show

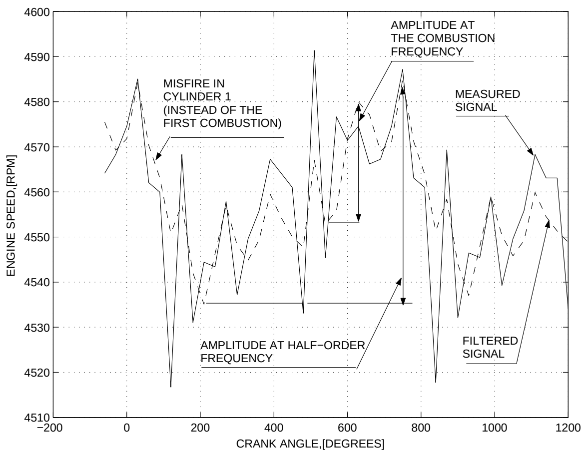

Fig. 2. Two engine cycles are plotted in the event of a misfire. Engine speed is 4500 [rpm]. Engine is operating at full load. The window size is $w = 20$. Measured engine speed signal is plotted with a solid line. Filtered signal with the filter (19) is plotted with dashed line.

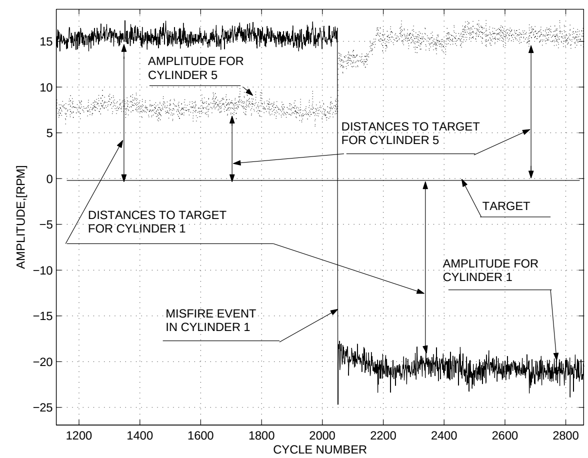

Fig. 3. Two amplitudes for the first and the fifth cylinders are plotted as a function of cycle number. Engine speed is 4500 [rpm]. The engine is operating at full load. The misfire is generated in the first cylinder. The amplitude for the first cylinder is plotted with a solid line. The amplitude for the fifth cylinder is plotted with dotted line. The differences between the target value the mean values of the amplitudes are indicated as distances to target.

that $F_j = F_{j-1}^2$ and hence $F_j = F_0^{2^j}$. If $\| F_0 \| \leq c < 1$, where c is a positive constant, then $\| F_j \| \leq c^{2^j}$. Hence $F_j \to 0$ as $j \to \infty$ and $\Gamma_{cj} \to [\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T]^{-1}$ as $j \to \infty$.

III. Filtering Technique Based on Trigonometric Interpolation Method

Filtering technique based on the trigonometric interpolation method can be divided in two steps. First, the engine speed is approximated via a trigonometric polynomial (1) in a window of a certain size moving in time. The performance of the approximation is determined by the error (6). Secondly, the filtered engine speed signal is defined by using two frequencies - combustion frequency and half-oder frequency, i.e.,

$$\omega_{fk} = \varphi_{fk}^T \theta_{fk}, \tag{19}$$

where θ_{fk} is the vector of the adjustable parameters

$$\theta_{fk}^T = [a_{0k} \quad a_{ck} \quad b_{ck} \quad a_{hk} \quad b_{hk}], \tag{20}$$

$$\varphi_k^T = [1 \quad cos(q_c x_k) \quad sin(q_c x_k) \quad cos(q_h x_k) \quad sin(q_h x_k)] \tag{21}$$

where ω_{fk} is the filtered engine speed signal, q_c and q_h are the combustion and half-oder frequencies respectively.

Figure 2 shows measured engine speed signal and filtered engine speed signal with filter (19).

IV. Statistical Misfire Detection Technique

A. *Misfire Detection at the Combustion Frequency*

The combustion state of a given cylinder is defined via the amplitude which is computed as a difference between maximal and minimal values of the engine speed for a cylinder whose power stroke occurs in the interval at the combustion frequency. Figure 3 shows the plots of the amplitudes for the first and the fifth cylinders, which produce two first combustions on the engine cycle (the firing sequence for a six cylinder engine is $1-5-3-6-2-4$). The misfire is generated in the first cylinder. The misfire can be detected by comparing the amplitude signals with a target value of the amplitude (for example with zero). The differences between the target value and the average values of the amplitudes are indicated as distances to target.

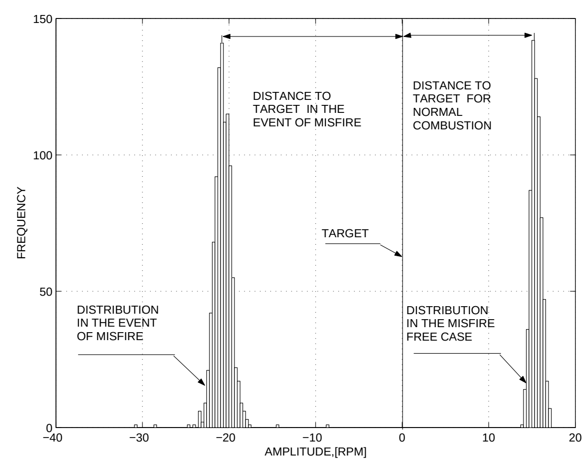

Fig. 4. Distributions of the amplitude signal for the first cylinder in the event of a misfire and in the misfire free case. The misfire is generated in the first cylinder. Engine speed is 4500 [rpm]. The engine is operating at full load. The sample size is 1624. The differences between the target value the mean values of the amplitudes are indicated as distances to target.

Figure 4 shows the distributions of the amplitude signal for the first cylinder in the event of a misfire in the first cylinder and in the misfire free case. The distributions in both cases are close to normal distributions with good separation between the mean and the target values.

1) One Sample T-test for Misfire Detection: Suppose that M amplitudes are computed at the combustion frequency as follows $A_i = \omega_{imax} - \omega_{imin}$, where ω_{imax} is the maximal value of the engine speed for cylinder i and ω_{imin} is a minimal value of the engine speed for cylinder i, $i = 1, ..., M$, where M is the number of engine cylinders (see Figure 1).

One Sample T-test is the statistical test for a comparison of a one sample average to a target value. Let A_l be the values of the amplitude to be examined, where l is the cycle number ($l \geq N$), N is the size of a moving window expressed in terms of a number of cycles where the amplitude is examined. Denoting a target value of the amplitude as a_t the null hypothesis is $H_0 : \overline{A}_l = a_t$, where $\overline{A}_l$, is the averaged value of the amplitude. Two alternative hypotheses are considered $H_{a1} : \overline{A}_l > a_t$ and $H_{a2} : \overline{A}_l < a_t$. The first alternative hypothesis indicates the misfire free case and the second one indicates a misfire. The algorithm for hypothesis testing can be divided in two steps.

Step 1. First the hypothesis is tested in the moving window of a minimal size N_{min}. The following $t - statistic$ is computed

$$t_l = \frac{\mid \overline{A}_l - a_t \mid \sqrt{N-1}}{s_l} \quad (22)$$

where $\overline{A}_l = \frac{1}{N} \sum_{i=l-(N-1)}^{i=l} A_i$, is the value of the amplitude averaged over the window of a size $N = N_{min}$, s_l is a standard deviation, $s_l = \sqrt{\frac{1}{N-1} \sum_{i=l-(N-1)}^{i=l} (A_i - \overline{A}_l)^2}$. Notice, that an average amplitude $\overline{A}_l$ and a standard deviation s_l can be computed recursively in each step of the moving window.

The value in the Student distribution look-up table for a certain significance level α and degrees of freedom $f = (N_{min} - 1)$ is compared with (22) in each step of the moving window. If the value of the statistic (22) is greater than the value in the Student distribution look-up table then the null hypothesis is rejected. If the average value of the amplitude is positive H_{a1} is accepted that indicates the misfire free case. In all other cases the misfire is indicated. If the value of the statistic calculated with (22) is less than the value in the Student distribution look-up table for a certain significance level α and degrees of freedom $f = (N_{min} - 1)$ the value of the statistic should be corrected by adding an additional degrees of freedom, i.e., increasing the window size.

Step 2. In this step the size of the moving window is increased until the value of the statistic (22) is greater than the value in the Student distribution look-up table. The Student distribution look-up table for a certain significance level is re-scaled so that for a certain sample size N the values correspond to the sample size $(N-1)$. Then the values in the re-scaled table are approximated by the following polynomial:

$$t_t = a_0 + \frac{a_1}{z} + \frac{a_2}{z^2} + \frac{a_3}{z^3} \quad (23)$$

where $z = \sqrt{N-1}$, a_i, $i = 0, ...3$ are the coefficients computed by using a least-squares curve fitting algorithm. The window size N_* is defined as a solution for a minimal N of the following equation:

$$\frac{\mid \overline{A}_l - a_t \mid z}{s_l} - (a_0 + \frac{a_1}{z} + \frac{a_2}{z^2} + \frac{a_3}{z^3}) = \delta \quad (24)$$

where δ is a small positive number. Equation (24) is the fourth order algebraic equation which can be solved by using standard numerical or analytical methods. The window size N_* which satisfies equation (24) guarantees that $t_l > t_t$ rejecting the null hypothesis and detecting a misfire, if any. Notice, that the window size N_* is bounded, i.e., $N_{min} < N_* \leq N_{max}$, where N_{max} is the maximal allowable window size (for example one can take $N_{max} = 100$). If a minimal solution of the equation (24) is larger than N_{max} the null hypothesis can not be tested and misfire can not be detected. Notice that, N_* is a predicted window size and the mean value and the variance of the amplitude signal might change when new data is coming into the window. Therefore Step 2 should be repeated when the window size reaches the value of N_* thereby further increasing of the window size should be made, if required. When the window size N_* which guarantees that $t_l > t_t$ is selected the misfire is detected at each step of the moving window. When the detection algorithm is deactivated the window size N_* is saved in the memory of the engine electronic control unit providing the updated value of the minimal window size in the first Step of the algorithm so that the next start of the detection algorithm begins with renewed value of the minimal window size. If the the window size N_* calculated as a solution of the equation (24) is greater than the maximal allowable size N_{max} the hypothesis can not be tested and hence the misfire can not be detected. This eventually means a drastic deterioration of the quality of the amplitude signal. The significance level α could be increased in this case reducing the values of t_t in the Student distribution look-up table to guarantee that $t_l > t_t$. Increasing of the significance level increases the probability of rejecting the null hypothesis mistakenly and hence the probability of erroneously detecting a misfire.

The algorithm described above is illustrated by two examples.

Example 1: New Engine. Consider the misfire event shown in Figure 3. Step 1 of the algorithm shows that the average value of the amplitude is $\overline{A}_l = -19.15[rpm]$, with a standard deviation $s_l = 0.62[rpm]$ for the window size of ten, $N_{min} = 10$. The value of the statistic is $t_l = 92.67$, with a zero target value $a_t = 0$. This value is greater than the value in the Student distribution look-up table for a significance level $\alpha = 0.0005$ and degrees of freedom 9 (see [1] page 251), $t_t = 4.781$. Therefore, the null hypothesis is rejected and since the average value of the amplitude is negative the misfire in the first cylinder is correctly detected.

Example 2: Aiged Engine.

The amplitude signal which is used for a misfire detection deteriorates due to the aging of the engine components. Deteriorated amplitude signal as a function of a cycle number

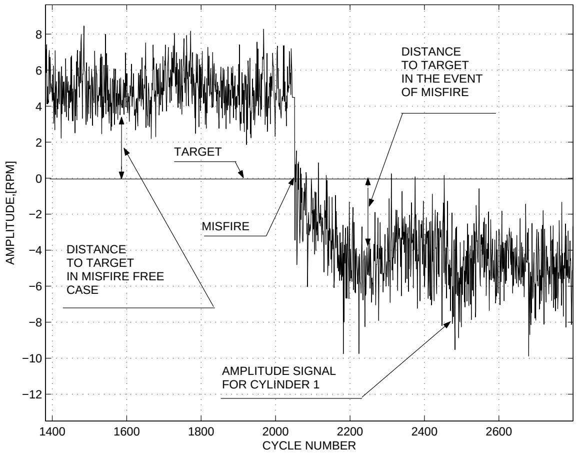

Fig. 5. The amplitude of the first cylinder is plotted as a function of a cycle number. Engine speed is 4500 [rpm]. The engine is operating at full load. The misfire is generated in the first cylinder. The difference between the target value the mean value of the amplitude is indicated as a distance to target.

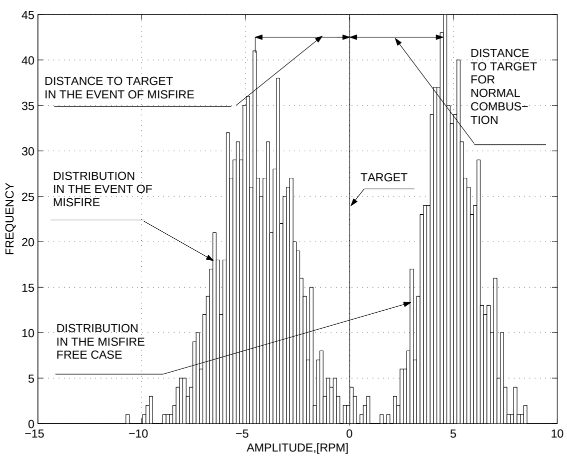

Fig. 6. Distributions of the amplitude signal for the first cylinder in the event of a misfire and in the misfire free case. The misfire is generated in the first cylinder. Engine speed is 4500 [rpm]. The engine is operating at full load. The sample size is 1624. The differences between the target value the mean values of the amplitudes are indicated as distances to target.

for aged engine is shown in Figure 5 (compare it with Figure 3). The distributions in the event of a misfire and for the misfire free case are shown in Figure 6.

Suppose that Step 1 of the algorithm shows the average amplitude $\overline{A}_l = -0.6519[rpm]$ with a standard deviation $s_l = 0.7436[rpm]$ for the window size of ten. The value of the statistic is $t_l = 2.63$, with a zero target value $a_t = 0$. This value is less than the value in the Student distribution look-up table for a significance level $\alpha = 0.0005$ and degrees of freedom 9 (see [1] page 251), $t_t = 4.781$. Therefore the next step is taken.

Step 2. Recursive application of the algorithm described above gives the following window size $N_* = 46$ with $\delta = 0.001$ and the average amplitude $\overline{A}_l = -0.6519[rpm]$ with a standard deviation $s_l = 1.24[rpm]$. The value of the statistic is $t_l = 3.5267$ while the value in the Student distribution look-up table for a significance level $\alpha = 0.0005$ and degrees of freedom 45 (see [1] page 251), $t_t = 3.522$. Therefore the null hypothesis is rejected and the misfire is detected.

B. *Misfire Detection at the Half-Order Frequency*

Another technique for a misfire detection is a monitoring of the amplitude at the half-order frequency (see Figure 1). The phase of the half-oder frequency component shows which cylinder is misfiring. All the amplitudes corresponding to all engine cylinders are calculated at the half-order frequency. In the misfire free case all the amplitude signals should be close to zero. In the event of a misfire the amplitudes deviate from zero and the misfire is suspected in the cylinder with the largest mean value of the amplitude. In the first stage of the detection the amplitude with the largest mean value is compared to the target value which should be chosen relatively large in order to separate the oscillations induced by the misfire event from the oscillations induced by other events. To this end the amplitude signal is statistically tested via *One Sample T-test* described above. In the second stage of the detection a mean value of the largest amplitude is compared to the mean value of the next largest amplitude by hypothesis testing of the equality of two means (*Two Sample T-test*) in order to detect which cylinder is misfiring. The test statistic is the $t - statistic$ where the hypothesis that the mean values of two amplitudes are equal is taken as a null hypothesis which indicates that the misfire is not recognizable. Alternative hypothesis that the mean value of one of the amplitudes is the largest indicates a misfire in the corresponding cylinder. *Two Sample T-test* for a misfire detection can be performed similarly to the *One Sample T-test* described above and therefore is omitted in the present paper.

V. CONCLUSIONS

New recursive filtering algorithms for misfire detection based on the trigonometric interpolation method proposed in the present paper improve the performance of the filtering technique allowing a flexible choice of the size of the moving window, and correction algorithms for trigonometric interpolation method ensure the robustness with respect to round-off errors which are always present in the finite precision implementation environment. Statistical decision making mechanism which is based on a hypothesis testing introduced in the present paper allows to make a misfire detection with a certain significance level with automatically selected sample size depending on the signal quality that in turn improves the robustness of the misfire detection method.

REFERENCES

[1] Black Belt Memory Jogger: A Pocket Guide for Six Sigma Success, D. Picard Editor, USA, GOAL/QPC, 2002.
[2] Rizzoni G., Guezennec Y., Soliman A., Lee B. (2005). Engine Control Using Torque Estimation *US Patent 6866024 B2*.
[3] Stotsky A. Computationally Efficient Filtering Algorithms for Engine Torque Estimation, Proc. IMechE, vol. 219, 2005, Part D: J. Automobile Engineering, pp. 1099-1107.