Statistical Engine Misfire Detection

A. Stotsky
Volvo Car Corporation
Engine Design and Development
Dept. 92545 HB1N
SE- 405 31 Gothenburg
Sweden
e-mail: astotsky@volvocars.com

OUTLINE

1 INTRODUCTION

- Present paper shows the application of modern signal processing technique for filtering and statistical tools for robust engine misfire detection.
- Misfire is the state of an engine where the combustion does not occur due to the errors in fueling or ignition.
- The misfiries cause changes in the crankshaft rate of rotation, because the misfired cylinder is not able to provide the torque.
- Low frequency oscillations from the powertrain and high frequency oscillations due to the crankshaft torsion, together with vibrations induced by the road, act as disturbances on the crankshaft at high rotational speeds.
- Recursive DFT (Discrete Fourier Transformation) method in the window of a certain size moving in time can be used for filtering at the engine firing frequency.

- Algorithms proposed in the present paper allow to make a trigonometric interpolation of the engine speed data for any window size.
- Trigonometric interpolation method proposed in this paper requires a matrix inversion, as it is usual for least-squares fitting, and this in turn, makes the method computationally expensive.
- Limited precision effects might severely deteriorate the performance of the recursive trigonometric interpolation method.
- The misfire detection approach proposed in the present paper is based on a monitoring of the amplitudes at two frequencies.
- Conventional detection techniques compare the amplitude at the combustion frequency to the threshold value.

- Amplitude signal becomes very noise contaminated due to the aging of the engine components.
- Another method for improvement of the misfire detectability of aged engines is introduction of the statistical decision making technology.
- The amplitudes are hypothesis tested for a misfire detection. *One Sample T-test* (the name is carried over from Six Sigma) which compares the average value of the amplitude to the target value is used for a misfire detection at the combustion frequency.
- The algorithm results in automatic selection of the window size (number of engine cycles) for new and aged engine that guarantees misfire detection with the same significance level.

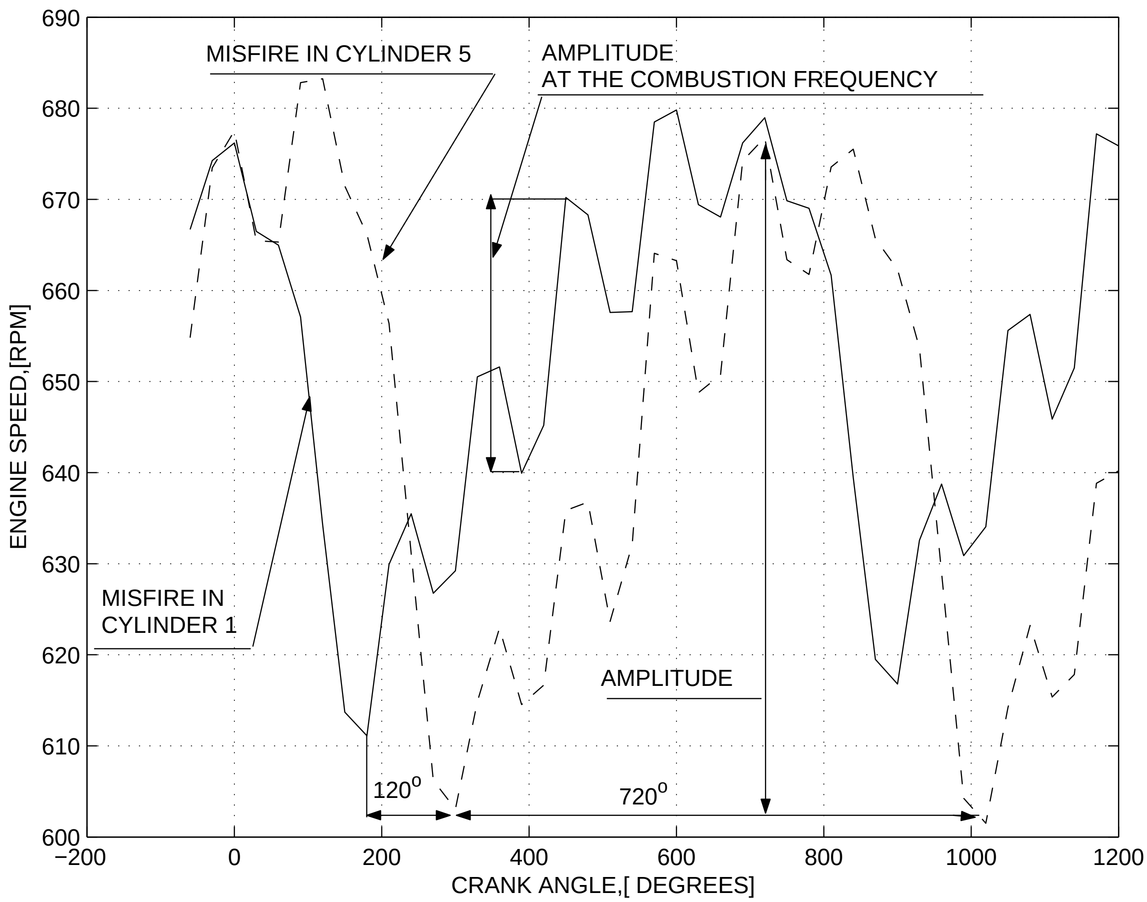

FIG 1: Two engine cycles are plotted in the events of a misfire in the fisrt and fifth cylinders. Engine is operating at idle. Engine speed signal in the event of a misfire in the first cylinder is plotted with a solid line. Engine speed signal in the event of a misfire in the fifth cylinder is plotted with a dashed line.

OUTLINE

2 RECURSIVE TRIGONOMETRIC INTERPOLATION ALGORITHMS

2.1 PROBLEM STATEMENT

Engine speed ω_k, $k = 1, 2...$ is measured at the following points $x_k = k\Delta$, where Δ is a step size.
The engine speed signal is approximated as follows:

$$\hat{\omega}_k = \varphi_k^T \theta_k, \tag{1}$$

$$\theta_k^T = [a_{0k} \quad a_{q1k} \quad b_{q1k} \quad a_{q2k} \quad b_{q2k}, \\ ..., a_{qnk} \quad b_{qnk}], \tag{2}$$

$$\varphi_k^T = [1 \quad cos(q_1 x_k) \quad sin(q_1 x_k) \quad cos(q_2 x_k) \\ sin(q_2 x_k), ..., cos(q_n x_k) \quad sin(q_n x_k)] \tag{3}$$

Measured engine speed signal is presented as follows:

$$\omega_k = \varphi_k^T \theta_*, \tag{4}$$

$$\theta_*^T = [a_{0*} \quad a_{q1*} \quad b_{q1*} \quad a_{q2*} \quad b_{q2*}, ..., a_{qn*} \quad b_{qn*}], \tag{5}$$

and a_{0*}, a_{q*} and b_{q*} are constant unknown coefficients.

Introducing a moving window of a size w the measured engine speed signal ω_k is approximated in the least squares sense.

The error to be minimized at every step is as follows:

$$E_k = \sum_{i=k-(w-1)}^{i=k} (\omega_i - \hat{\omega}_i)^2, \qquad k \geq w \tag{6}$$

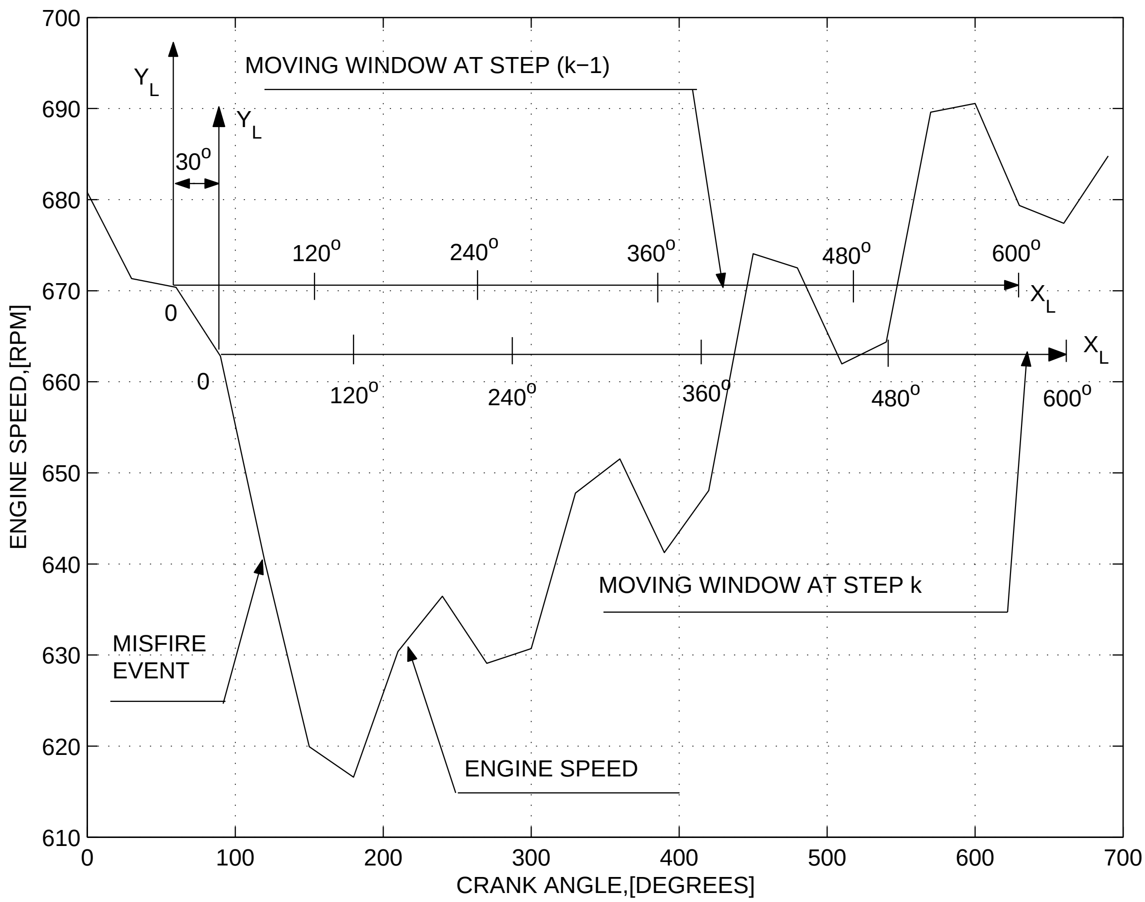

FIG 2: Measurements with a step of 30^o on six cylinder prototype engine. Misfire is generated in the first cylinder. The engine speed is plotted with a solid line. The engine is operating at idle. A window of a size $w = 20$ moving in time is defined in the form of local coordinates X_L and Y_L.

2.2 RECURSIVE ALGORITHMS FOR TRIGONOMETRIC INTERPOLATION

The vector of adjustable parameters θ_k:

$$\theta_k = [\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T]^{-1} \sum_{i=k-(w-1)}^{i=k} \varphi_i \omega_i \tag{7}$$

The vector of adjustable parameters at step $k-1$:

$$\theta_{k-1} = [\sum_{i=k-w}^{i=k-1} \varphi_i \varphi_i^T]^{-1} \sum_{i=k-w}^{i=k-1} \varphi_i \omega_i \tag{8}$$

The vector of adjustable parameters θ_k can be presented as follows:

$$\begin{aligned}\theta_k \;=\; & [(\sum_{i=k-w}^{i=k-1} \varphi_i \varphi_i^T) - \varphi_{k-w} \varphi_{k-w}^T + \varphi_k \varphi_k^T]^{-1} \\ & [(\sum_{i=k-w}^{i=k-1} \varphi_i \omega_i) + \varphi_k \omega_k - \varphi_{k-w} \omega_{k-w}] \end{aligned} \tag{9}$$

$$\theta_{rk} = (I - \frac{A_{k-1} \varphi_k \varphi_k^T}{1 + \varphi_k A_{k-1} \varphi_k^T}$$

$$(\theta_{r(k-1)} + \frac{\Gamma_{k-1}\varphi_{k-w}\varphi_{k-w}^T\theta_{r(k-1)}}{1-\varphi_{k-w}^T\Gamma_{k-1}\varphi_{k-w}}))$$
$$+\Gamma_k(\varphi_k\omega_k - \varphi_{k-w}\omega_{k-w}) \quad (10)$$

where

$$\Gamma_k = A_{k-1}(I - \frac{\varphi_k\varphi_k^T A_{k-1}}{1+\varphi_k A_{k-1}\varphi_k^T}) \quad (11)$$
$$A_{k-1} = \Gamma_{k-1}(I + \frac{\varphi_{k-w}\varphi_{k-w}^T\Gamma_{k-1}}{1-\varphi_{k-w}^T\Gamma_{k-1}\varphi_{k-w}}) \quad (12)$$

The elements of the regressor φ_i, $i \geq 3$ are recursively calculated via the following Chebyshev's three term recurrence relations

$$\varphi_i = 2d_q \quad * \quad \varphi_{i-1} - \varphi_{i-2} \quad (13)$$
$$\varphi_i^T = [1 \quad cos(q_1 i\Delta) \quad sin(q_1 i\Delta) \quad cos(q_2 i\Delta)$$
$$sin(q_2 i\Delta), ..., cos(q_n i\Delta) \quad sin(q_n i\Delta)] \quad (14)$$
$$d_q^T = [1 \quad cos(q_1\Delta) \quad cos(q_1\Delta) \quad cos(q_2\Delta)$$
$$cos(q_2\Delta), ..., cos(q_n\Delta) \quad cos(q_n\Delta)] \quad (15)$$

2.3 CORRECTION OF THE RECURSIVE ALGORITHMS FOR ROUND-OFF ERRORS

• On-board implementation of the recursive trigonometric interpolation algorithms is done in ECU (Engine Control Unit) in the finite precision environment, where round-off errors are recursively accumulated.

• This makes the recursive trigonometric interpolation algorithm unsuitable for continuous use without correction. The parameter vector θ_{rk} and the inverse of the 'information matrix' Γ_k obtained by using the recursive least-squares algorithm are used as the initial states for the correction algorithms.

2.3.1 CORRECTION OF θ_{rk}

Consider the following algorithm for correction of θ_{rk}.

$$\theta_{cj} = \theta_{c(j-1)} - \Gamma_k e_{j-1} \tag{16}$$

where θ_{cj} is the correction of the parameter vector θ_{rk},

$$e_{j-1} = (\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T)\theta_{c(j-1)} - \sum_{i=k-(w-1)}^{i=k} \varphi_i \omega_i, \tag{17}$$

where $\theta_{c0} = \theta_{rk}$.

The estimation error $\tilde{\theta}_{cj} = \theta_{cj} - \theta_k$ is the following :

$$\tilde{\theta}_{cj} = (I - \Gamma_k[\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T])\tilde{\theta}_{cj-1} \tag{18}$$

Γ_k is a recursive estimate of $[\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T]^{-1}$.

$\tilde{\theta}_{cj} \rightarrow 0$ as $j \rightarrow \infty$ if the eigenvalues of the following matrix $(I - \Gamma_k [\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T])$ are located inside of the unit circle.

2.3.2 CORRECTION OF Γ_k

Consider the following algorithm for correction Γ_k:

$$\Gamma_{cj} = \Gamma_{c(j-1)} + \Gamma_{c(j-1)} F_{j-1} \tag{19}$$

where Γ_{cj} is the correction of the matrix Γ_k,

$$F_{j-1} = I - [\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T] \Gamma_{c(j-1)} \tag{20}$$

with the initial condition $\Gamma_{c0} = \Gamma_k$, $j = 1, 2, ...$

Straightforward calculations show that $F_j = F_{j-1}^2$ and hence $F_j = F_0^{2^j}$. If $\| F_0 \| \leq c < 1$, where c is a positive constant, then $\| F_j \| \leq c^{2^j}$. Hence $F_j \to 0$ as $j \to \infty$ and $\Gamma_{cj} \to [\sum_{i=k-(w-1)}^{i=k} \varphi_i \varphi_i^T]^{-1}$ as $j \to \infty$.

3 Filtering Technique Based on Trigonometric Interpolation Method

Filtered engine speed signal is defined by using two frequencies - combustion frequency and half-oder frequency, i.e.,

$$\omega_{fk} = \varphi_{fk}^T \theta_{fk}, \tag{21}$$

where θ_{fk} is the vector of the adjustable parameters

$$\theta_{fk}^T = [a_{0k} \quad a_{ck} \quad b_{ck} \quad a_{hk} \quad b_{hk}], \tag{22}$$

$$\varphi_k^T = [1 \quad cos(q_c x_k) \quad sin(q_c x_k) \quad cos(q_h x_k) \quad sin(q_h x_k)] \tag{23}$$

where ω_{fk} is the filtered engine speed signal.

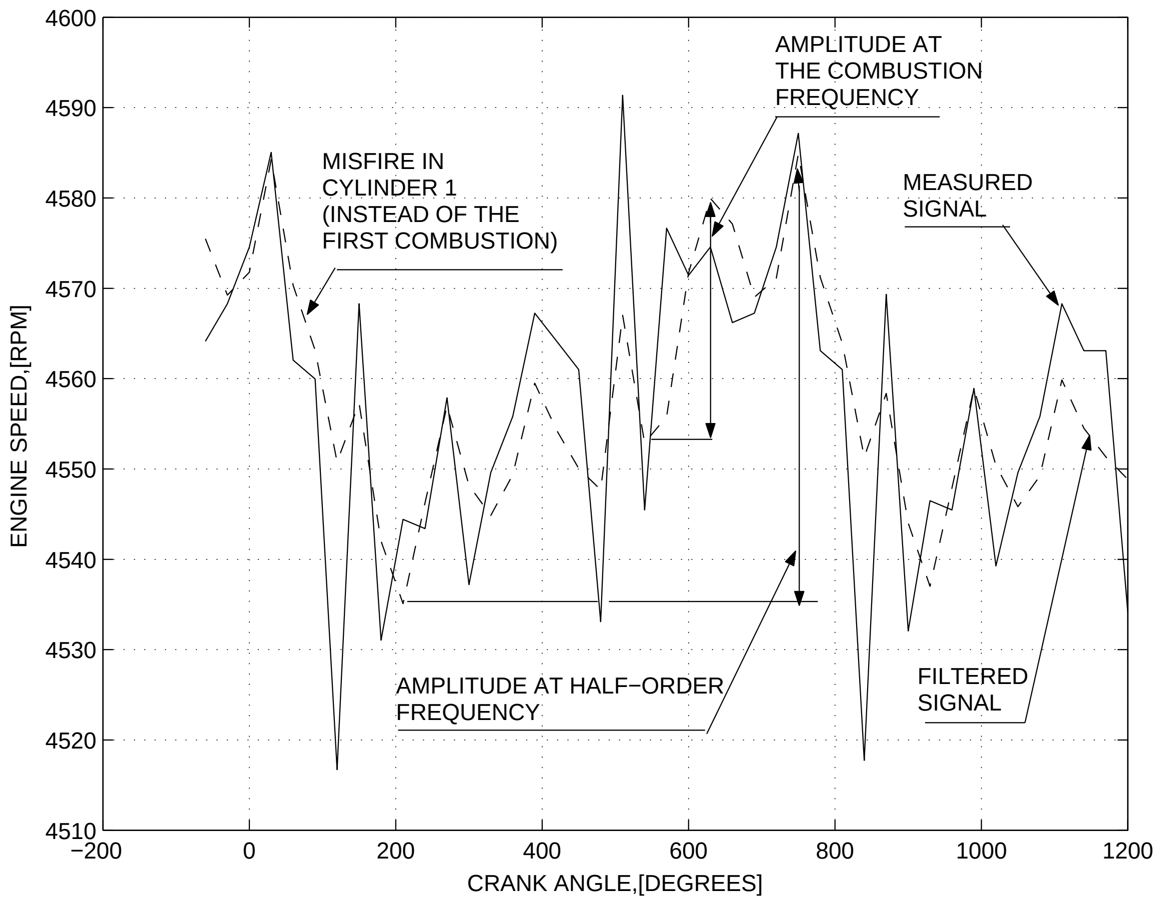

FIG 3: Two engine cycles are plotted in the event of a misfire. Engine speed is 4500 [rpm]. Engine is operating at full load. The window size is $w = 20$. Measured engine speed signal is plotted with a solid line. Filtered signal with the filter (21) is plotted with dashed line.

OUTLINE

1. Introduction

2. Recursive Trigonometric Interpolation Algorithms

2.1 Problem Statement

2.2 Recursive Algorithms for Trigonometric Interpolation

2.3 Correction of the Recursive Algorithms for Round-Off Errors

3. Filtering Technique Based on Trigonometric Interpolation Method

$\surd$ 4. Statistical Misfire Detection Technique

4.1 *One Sample t-Test* for Misfire Detection

4.2 Example 1: New Engine

4.3 Example 2: Old Engine

5. Conclusions

4 STATISTICAL MISFIRE DETECTION TECHNIQUE

• The misfire detection is associated with the hypothesis test. The amplitude signal at the combustion frequency is statistically tested via *One Sample T-test* which compares the average amplitude to a target value.

• For hypothesis testing the value of $t-statistic$, in the window moving in time, is compared to the value in the Student distribution look-up table for a certain significance level and degrees of freedom (size of the moving window).

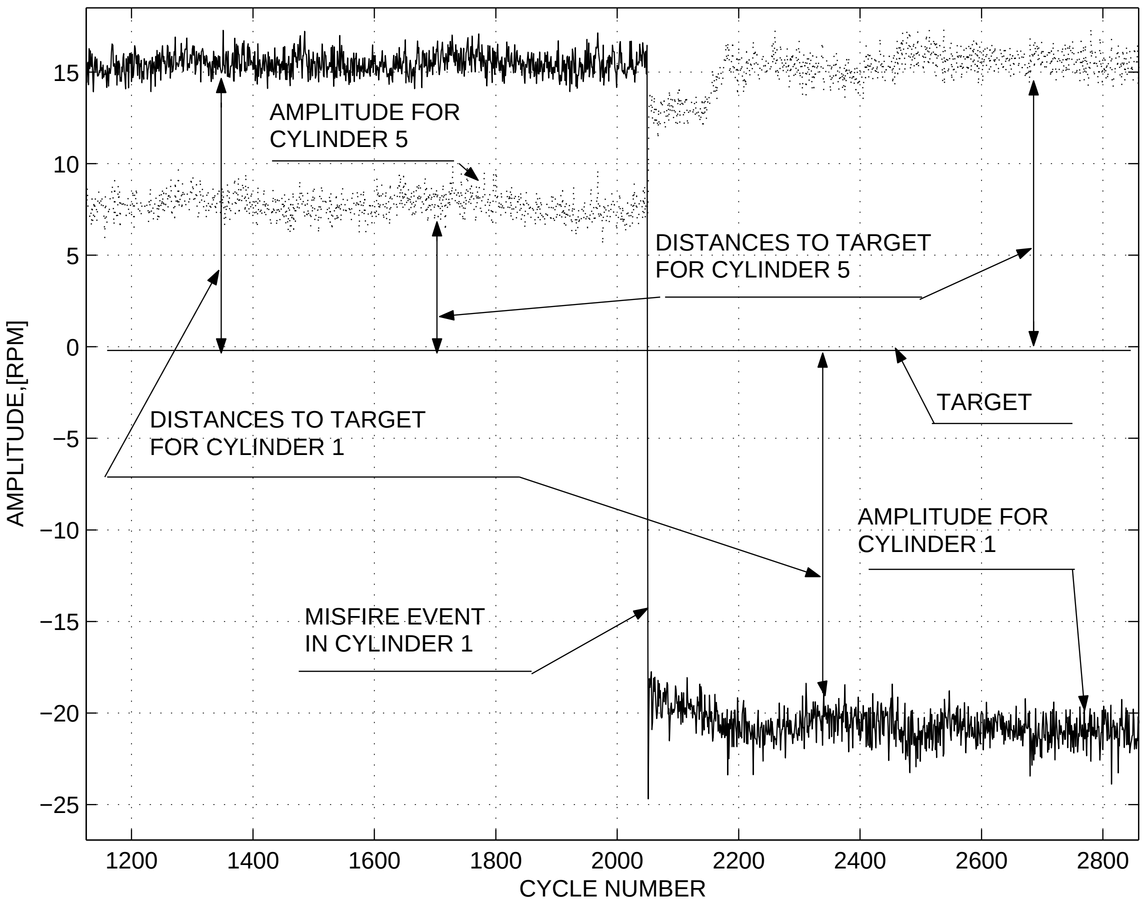

FIG 4: Two amplitudes for the first and the fifth cylinders are plotted as a function of cycle number. Engine speed is 4500 [rpm]. The engine is operating at full load. The misfire is generated in the first cylinder. The amplitude for the first cylinder is plotted with a solid line. The amplitude for the fifth cylinder is plotted with dotted line. The differences between the target value the mean values of the amplitudes are indicated as distances to target.

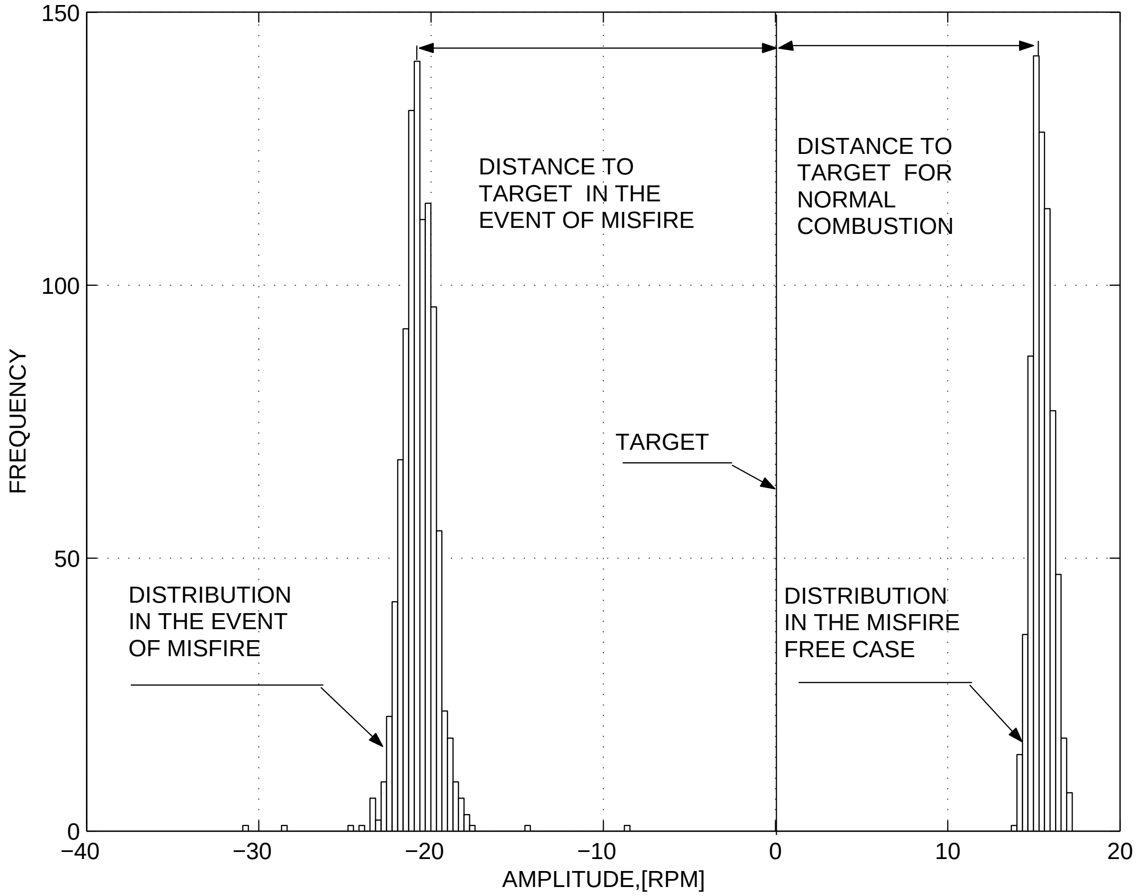

FIG 5: Distributions of the amplitude signal for the first cylinder in the event of a misfire and in the misfire free case. The misfire is generated in the first cylinder. Engine speed is 4500 [rpm]. The engine is operating at full load. The sample size is 1624. The differences between the target value the mean values of the amplitudes are indicated as distances to target.

4.1 *ONE SAMPLE T-TEST* FOR MISFIRE DETECTION

Amplitudes: $A_i = \omega_{imax} - \omega_{imin}$, for cylinder i, $i = 1, ..., M$, are calculated.

The null hypothesis is $H_0 : \overline{A}_l = a_t$,

Two alternative hypotheses are considered $H_{a1} : \overline{A}_l > a_t$ and $H_{a2} : \overline{A}_l < a_t$.

Definition: The misfire is detectable if the alternative hypothesis $H_{a2} : \overline{A}_l < a_t$ is accepted.

The algorithm for hypothesis testing can be divided in two steps.

Step 1. Moving window of a minimal size N_{min}. The following $t - statistic$ is computed in each step:

$$t_l = \frac{\mid \overline{A}_l - a_t \mid \sqrt{N-1}}{s_l} \qquad (24)$$

where $\overline{A}_l = \dfrac{1}{N} \sum\limits_{i=l-(N-1)}^{i=l} A_i$, is the value of the amplitude averaged over the window of a size $N = N_{min}$, s_l is a standard deviation,

$s_l = \sqrt{\frac{1}{N-1}\sum_{i=l-(N-1)}^{i=l}(A_i - \overline{A}_l)^2}$. An average amplitude $\overline{A}_l$ and a standard deviation s_l are computed recursively in each step:

$$\begin{aligned} \overline{A}_l &= \overline{A}_{l-1} + \frac{1}{N}(A_l - A_{l-N}) \qquad (25) \\ s_l^2 &= s_{l-1}^2 + \frac{1}{N-1}[(A_l - \overline{A}_l)^2 \\ &- \frac{1}{N-1}(A_{l-N} - \overline{A}_l)^2] \qquad (26) \end{aligned}$$

Step 2. Selection of the window size.

The Student distribution look-up table for a certain significance level is approximated by the following polynomial:

$$t_t = a_0 + \frac{a_1}{z} + \frac{a_2}{z^2} + \frac{a_3}{z^3} \qquad (27)$$

where $z = \sqrt{N-1}$, a_i, $i = 0, ...3$ are the coefficients computed by using a least-squares curve fitting algorithm. The window size N_* is defined as a solution for a minimal N of the following equation:

$$\frac{\mid \overline{A}_l - a_t \mid z}{s_l} - (a_0 + \frac{a_1}{z} + \frac{a_2}{z^2} + \frac{a_3}{z^3}) = \delta$$

When the window size N_* which guarantees that $t_l > t_t$ is selected the misfire is detected at each step of the moving window.

The average value of the amplitude is $\overline{A}_l = -19.15[rpm]$,

The standard deviation is $s_l = 0.62[rpm]$ for the window size of ten, $N_{min} = 10$.

The value of the statistic is $t_l = 92.67$, with a zero target value $a_t = 0$.

The value in the Student distribution look-up table for a significance level $\alpha = 0.0005$ and degrees of freedom 9, $t_t = 4.781$.

The null hypothesis is rejected and the misfire in the first cylinder is correctly detected.

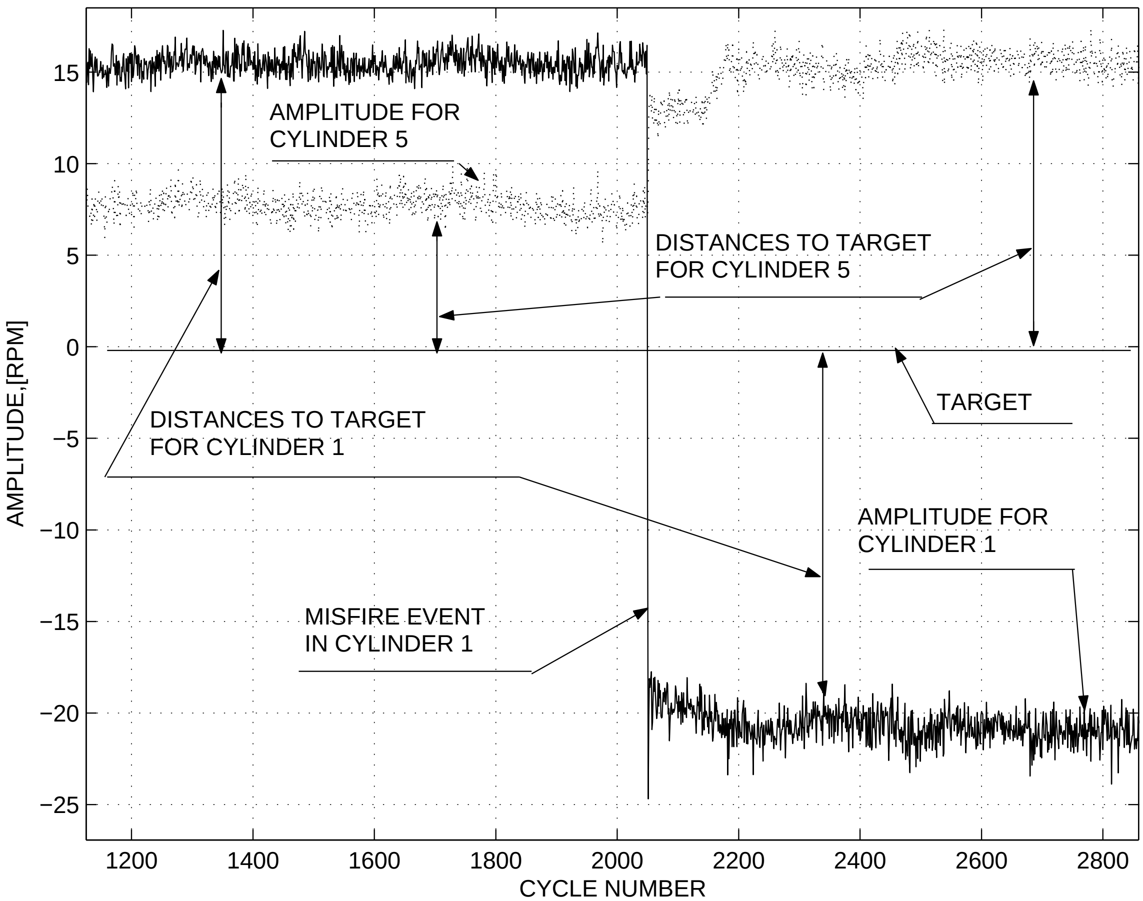

FIG 6: Two amplitudes for the first and the fifth cylinders are plotted as a function of cycle number. Engine speed is 4500 [rpm]. The engine is operating at full load. The misfire is generated in the first cylinder. The amplitude for the first cylinder is plotted with a solid line. The amplitude for the fifth cylinder is plotted with dotted line. The differences between the target value the mean values of the amplitudes are indicated as distances to target.

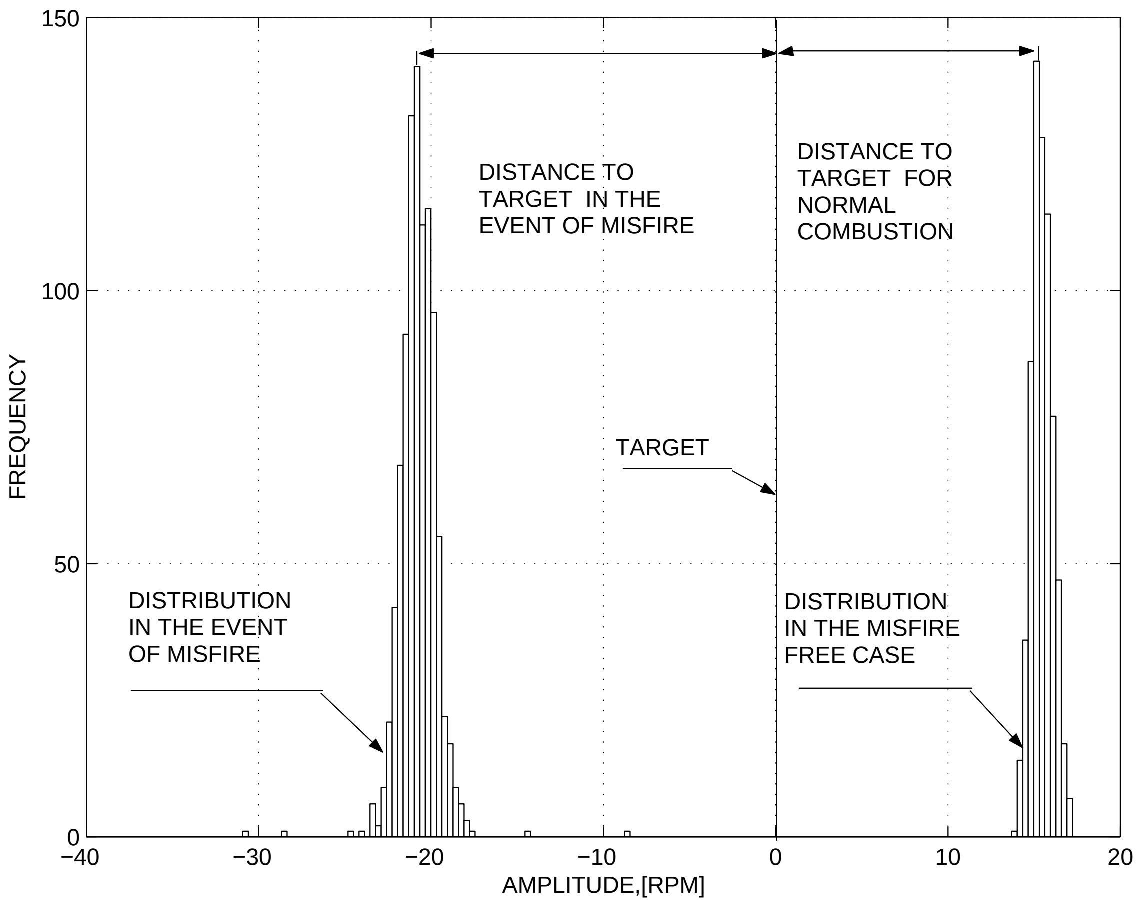

FIG 7: Distributions of the amplitude signal for the first cylinder in the event of a misfire and in the misfire free case. The misfire is generated in the first cylinder. Engine speed is 4500 [rpm]. The engine is operating at full load. The sample size is 1624. The differences between the target value the mean values of the amplitudes are indicated as distances to target.

4.3 EXAMPLE 2: AGED ENGINE

Average amplitude is $\overline{A}_l = -0.6519[rpm]$ in step 1.

Standard deviation is $s_l = 0.7436[rpm]$ for the window size of ten.

The value of the statistic is $t_l = 2.63$, with a zero target value $a_t = 0$.

The value in the Student distribution look-up table for a significance level $\alpha = 0.0005$ and degrees of freedom 9 is $t_t = 4.781$.

Step 2. The application of the algorithm gives the window size $N_* = 46$ with $\delta = 0.001$

The value of the statistic is $t_l = 3.5267$

The value in the Student distribution look-up table for a significance level $\alpha = 0.0005$ and degrees of freedom 45 is $t_t = 3.522$. Therefore the null hypothesis is rejected and the misfire is detected.

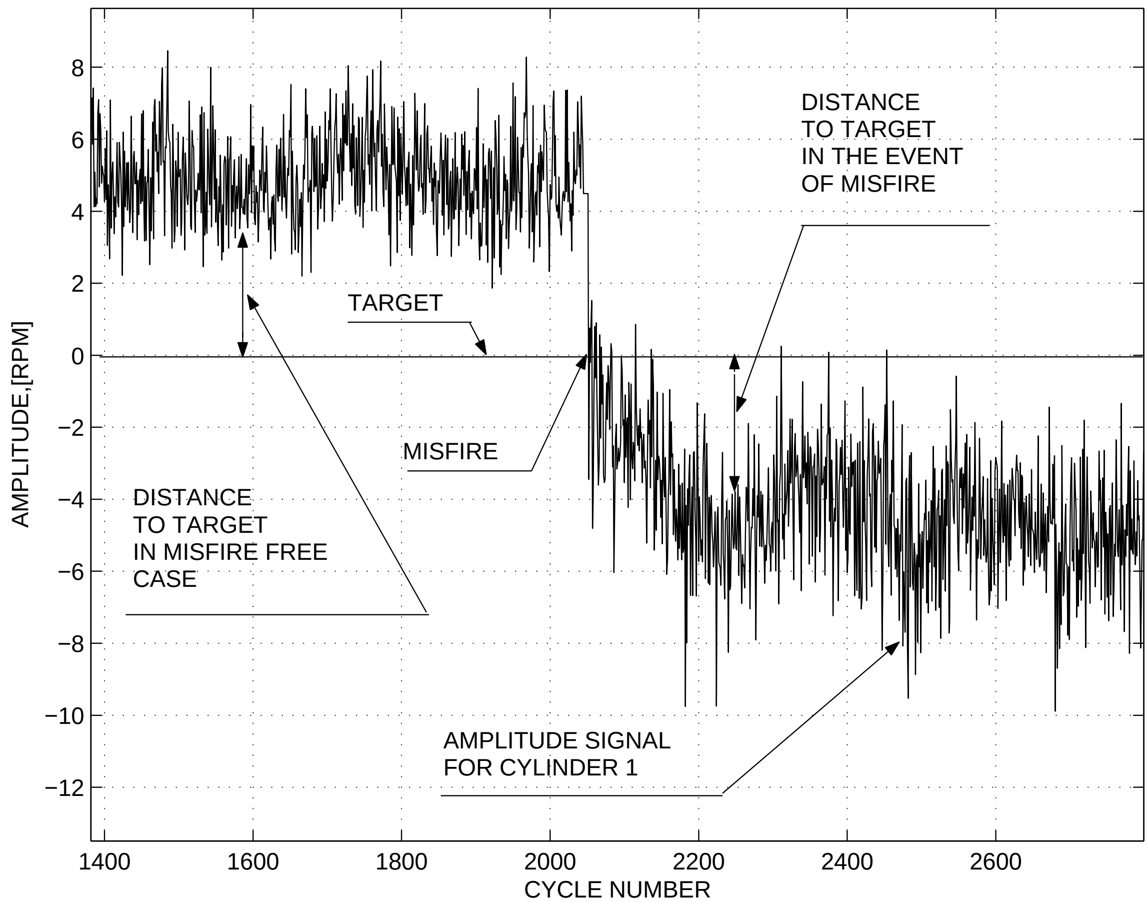

FIG 8: The amplitude of the first cylinder is plotted as a function of a cycle number. Engine speed is 4500 [rpm]. The engine is operating at full load. The misfire is generated in the first cylinder. The difference between the target value the mean value of the amplitude is indicated as a distance to target.

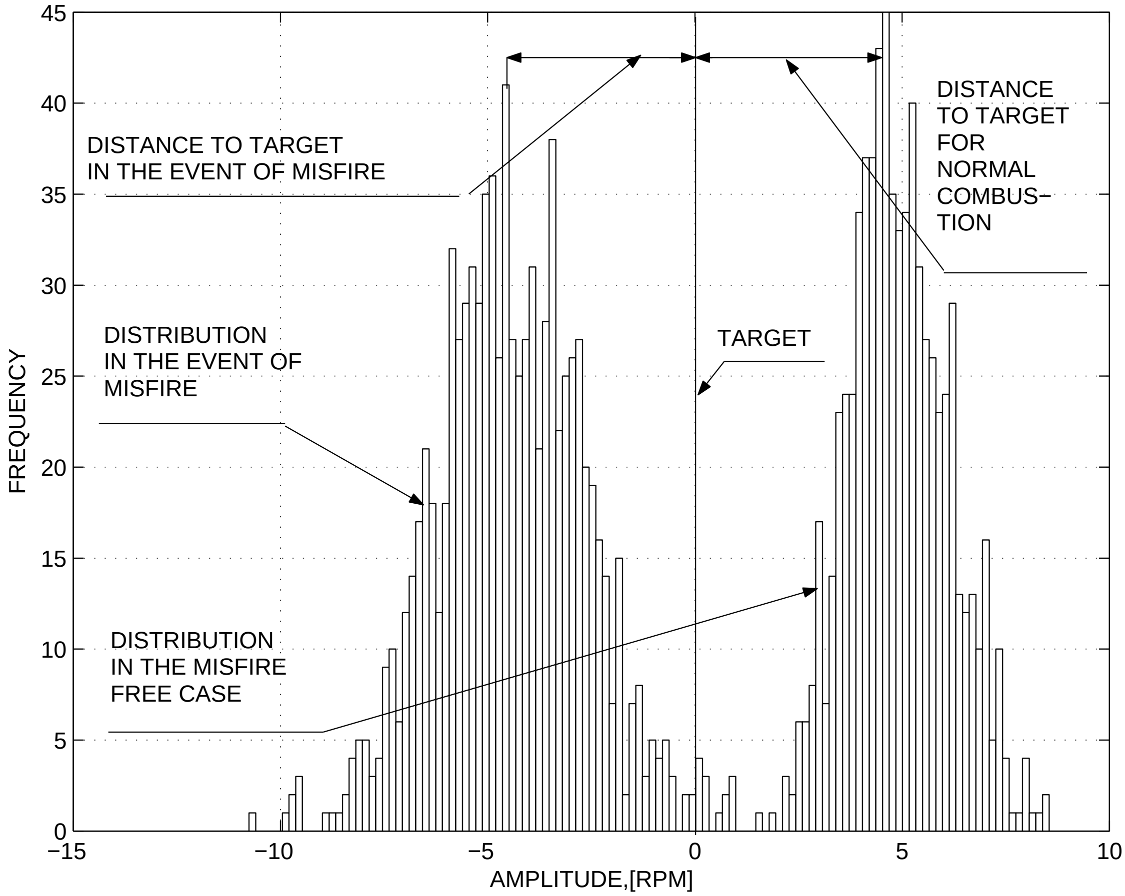

FIG 9: Distributions of the amplitude signal for the first cylinder in the event of a misfire and in the misfire free case. The misfire is generated in the first cylinder. Engine speed is 4500 [rpm]. The engine is operating at full load. The sample size is 1624. The differences between the target value the mean values of the amplitudes are indicated as distances to target.

5 CONCLUSIONS

• New recursive filtering algorithms for misfire detection based on the trigonometric interpolation method proposed in the present paper improve the performance of the filtering technique allowing a flexible choice of the size of the moving window, and correction algorithms for trigonometric interpolation method ensure the robustness with respect to round-off errors which are always present in the finite precision implementation environment.

• Statistical decision making mechanism which is based on a hypothesis testing introduced in the present paper allows to make a misfire detection with a certain significance level with automatically selected sample size depending on the signal quality that in turn improves the robustness of the misfire detection method.